Math 6051/3051: Recitation 2

Naufil Sakran

Do any **four** of the following problems.

(1) Use induction to prove

$$|a_1 + a_2 + \cdots + a_n| \leq |a_1| + |a_2| + \cdots + |a_n|$$

for all $a_1, \cdots, a_n \in \mathbb{R}$.

(2) For all $a, b \in \mathbb{R}$, prove that

$$||a| - |b|| \leq |a - b|.$$

(3) Show $[3 + \sqrt{2}]^{2/3}$ is not a rational number.

(4) Prove that if $a > 0$, then there exists $n \in \mathbb{N}$ such that $\frac{1}{n} < a < n$.

(5) For each set below that is bounded above, find the supremum and infimum for the set. Otherwise, write "DNE".

(a)
$$\bigcap_{n=1}^{\infty} \left\{ 1 - \frac{1}{n}, 1 + \frac{1}{n} \right\}.$$

(b)
$$\left\{ \frac{1}{n} : \text{ where } n \text{ is prime} \right\}$$

(c)
$$\left\{ n + \frac{(-1)^n}{n} : n \in \mathbb{N} \right\}$$

(d)
$$\{\cos x + \sin x : x \in \mathbb{Q}\}$$