Math 6051/3051: Recitation 3

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Do any **four** of the following problems.

(1) Let S and T be two non-empty bounded subsets of $\mathbb{R}$ such that $S \subseteq T$. Prove the following inequalities:

$$\inf T \leq \inf S \leq \sup S \leq \sup T$$

(2) Let A and B be nonempty bounded subsets of $\mathbb{R}$, and let $A + B$ be the set of all sums $a + b$ where $a \in A$ and $b \in B$. Prove that $\inf(A + B) = \inf(A) + \inf(B)$.

(3) Give an example of a sequence of irrational numbers having a limit $\lim x_n$ that is a rational number.

(4) Find the limits of the following sequence if they exist, otherwise write DNE. No proofs are required.

(a) $a_n = \frac{7n^3+8n}{3-2n^3}$.

(b) $a_n = \cos\left(\frac{2\pi}{3n}\right)$.

(c) $a_n = \frac{2^{n+1}+5}{2^n-7}$

(5) Determine the limit $\lim s_n$ where $s_n = \sqrt{n^2+1} - n$. No proofs are required, but show any relevant algebra.