

Math 6091/3091: Recitation 1

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Do any **all** of the following problems.

- (1) **(2 points)** Please state whether the following sets with their respective operations forms a group or not. If they do not form a group, please indicate which group axiom do they violate.

(a) The set of rational numbers with respect to addition $(\mathbb{Q}, +)$.

(b) The set of positive rational numbers with respect to multiplication $(\mathbb{Q}_{>0}, +)$.

(c) The set of rational numbers with respect to operation $*$ where $*$ is defined as: for $a, b \in \mathbb{Q}$,
 $a * b = \frac{a+b}{5}$.

(d) The set of real numbers with respect to the operation $*$ where $*$ is defined as: for $a, b \in \mathbb{R}$,
 $a * b = \min\{a, b\}$.

(e) The set of real numbers with respect to the operation $*$ where $*$ is defined as: for $a, b \in \mathbb{R}$,
 $a * b = a + b + ab$.

- (2) **(3 points)** Let $G = \{a \in \mathbb{R} \mid 0 \leq a < 1\}$ and for $a, b \in G$, let $a * b$ be the fractional part of $a + b$. For example

$$0.3 * 0.9 = 0.2, \quad 0.4 * 0.22 = 0.62, \quad 0.7 * 0.555 = 0.255, \quad 0.3 * 0.7 = 0.$$

(a) Show that $(G, *)$ forms an abelian group.

(b) Solve the equation $0.3 * X = 0.8$.

(3) **(2 points)** Please state whether the following sets with their respective operations forms a **ring** or not. If they do not form a ring, please indicate which axiom do they violate.

(a) The set $(P(x), +, \cdot)$ where $P(x)$ denote the set of polynomials in the variable x , $+$ denote usual polynomial addition and $\cdot$ denote usual polynomial multiplication.

(b) The set $(2\mathbb{Z}, +, \cdot)$ where $2\mathbb{Z}$ denote the set of even integers, $+$ denote usual addition and $\cdot$ denote usual multiplication.

(4) **(3 points)** Let $L(x) = \{\frac{f(x)}{g(x)} \mid f(x), g(x) \in P(x)\}$ where $P(x)$ is the denote the set of set of polynomials in the variable x . Define the usual addition and multiplication operation on $L(x)$.

(a) Show that $(L(x), +, \cdot)$ forms a field.

(b) Solve the equation

$$\frac{3x+2}{5x^2+3x+1}X - \frac{x^3}{x+2} = \frac{9x+5}{x^{10}}.$$

for X .