Math 6091/3091: Recitation 4

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Do any **all** of the following problems.

(1) **(3 points)** Let $V = \mathbb{R}^5$ and $W = \mathbb{R}^3$, and let $T : V \longrightarrow W$ be a linear transformation defined as

$$T((v_1, v_2, v_3, v_4, v_5)) = (v_1 - v_2, v_2 - v_3 + v_4, v_1 + v_2 - v_3 + 2v_4 + 3v_5).$$

(a) **(1 point)** Find the matrix representation of T.

Sol:

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 1 & 0 \\ 1 & 1 & -1 & 2 & 3 \end{bmatrix} \tag{1}$$

(b) **(2 point)** Find the bases for the kernel of T. Is T onto?

Sol:

$$T((v_1, v_2, v_3, v_4, v_5)) = 0$$
$$(v_1 - v_2, v_2 - v_3 + v_4, v_1 + v_2 - v_3 + 2v_4 + 3v_5) = (0, 0, 0)$$

From the first and second coordinate, we have $v_1 = v_2$ and $v_1 = v_2 = v_3 - v_4$. Putting in the last coordinate, we have $2v_1 = v_3 - 2v_4 - 3v_5 \implies v_1 = -v_4 - 3v_5$. So the kernel is

$$Ker(T) = span\{(1, 1, 0, -1, 0), (3, 3, 3, 0, -1)\}$$

(2) **(2 points)** Solve the following:

(a) Find the inverse of the following matrix:

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 4 & 1 & 0 \\ 4 & 0 & 4 \end{bmatrix}$$

Sol:

$$\begin{bmatrix} 1 & 1 & 2 & 1 & 0 & 0 \\ 4 & 1 & 0 & 0 & 1 & 0 \\ 4 & 0 & 4 & 0 & 0 & 1 \end{bmatrix} \implies \begin{bmatrix} 1 & 0 & 0 & -\frac{1}{5} & \frac{1}{5} & \frac{1}{10} \\ 0 & 1 & 0 & \frac{4}{5} & \frac{1}{5} & -\frac{2}{5} \\ 0 & 0 & 1 & \frac{1}{5} & -\frac{1}{5} & \frac{3}{20} \end{bmatrix}$$

So,

$$A^{-1} = \begin{bmatrix} -\frac{1}{5} & \frac{1}{5} & \frac{1}{10} \\ \frac{4}{5} & \frac{1}{5} & -\frac{2}{5} \\ \frac{1}{5} & -\frac{1}{5} & \frac{3}{20} \end{bmatrix}$$

(b) Compute $(A^{-1})^T * C$ where

$$C = \begin{bmatrix} 2 & 4 & 2 \\ 4 & 5 & 8 \\ 0 & 1 & 0 \end{bmatrix}$$

Sol:

$$\begin{bmatrix} -\frac{1}{5} & \frac{1}{5} & \frac{1}{10} \\ \frac{4}{5} & \frac{1}{5} & -\frac{2}{5} \\ \frac{1}{5} & -\frac{1}{5} & \frac{3}{20} \end{bmatrix}^T \begin{bmatrix} 2 & 4 & 2 \\ 4 & 5 & 8 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 2.8 & 3.4 & 6 \\ 1.2 & 1.6 & 2 \\ -1.4 & -1.45 & -3 \end{bmatrix}$$

(3) **(5 points)** Let $V = \mathbb{R}^3$ and $W = \mathbb{R}^3$, and let $T : V \longrightarrow W$ be a linear transformation defined as

$$T\left(\begin{bmatrix} 1 \\ 4 \\ 4 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 4 \\ 2 \end{bmatrix}, \qquad T\left(\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 4 \\ 5 \\ 8 \end{bmatrix}, \qquad T\left(\begin{bmatrix} 2 \\ 0 \\ 4 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}.$$

(a) **(4 point)** Find the matrix representation of T with respect to the standard basis.

Sol:

$$\begin{bmatrix} 2.8 & 1.2 & -1.4 \\ 3.4 & 1.6 & -1.45 \\ 6 & 2 & -3 \end{bmatrix}$$

(b) **(1 point)** Using the matrix, find $T(20, 20, 20)$.

Sol:

$$\begin{bmatrix} 2.8 & 1.2 & -1.4 \\ 3.4 & 1.6 & -1.45 \\ 6 & 2 & -3 \end{bmatrix} * \begin{bmatrix} 20 \\ 20 \\ 20 \end{bmatrix} = \begin{bmatrix} 52 \\ 71 \\ 100 \end{bmatrix}$$