Math 6091/3091: Recitation 7

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Do any **all** of the following problems.

(1) **(3 points)** Compute the determinant of the following matrices.

(a) **(1 point)**

$$\begin{bmatrix} 1 & 2 & 0 \\ 1 & 3 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

Sol:

$$\begin{vmatrix} 1 & 2 & 0 \\ 1 & 3 & 4 \\ 3 & 2 & 1 \end{vmatrix} = 17$$

(b) **(2 point)**

$$\begin{bmatrix} 1 & 3 & 2 & 1 \\ 3 & 2 & 1 & 0 \\ 1 & 7 & 3 & 4 \\ 1 & 1 & 0 & 4 \end{bmatrix}$$

Sol:

$$\begin{vmatrix} 1 & 3 & 2 & 1 \\ 3 & 2 & 1 & 0 \\ 1 & 7 & 3 & 4 \\ 1 & 1 & 0 & 4 \end{vmatrix} = 45$$

(2) **(2 points)** If A is an $n \times n$ matrix and B is obtained from A by multiplying each row of A by its row number, then show that

$$\det(B) = \frac{n(n+1)}{2} \det(A).$$

Sol:

$$\det(B) = 1\det(A) + 2\det(A) + \cdots + n\det(A) = (1 + 2 + \cdots + n)\det(A) = \frac{n(n+1)}{2}\det(A)$$

(3) **(3 points)** Use Cramer Rule to find the solution of the system of equations.

(a)

$$\begin{aligned} 7x_1 - 2x_2 &= 3 \\ 3x_1 + x_2 &= 5 \end{aligned}$$

Sol:

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

(b)

$$\begin{aligned} x - 4y + z &= 6 \\ 4x - y + 2z &= -1 \\ 2x + 2y - 3z &= -20 \end{aligned}$$

Sol:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{-1}{55} \begin{bmatrix} 144 \\ 61 \\ -230 \end{bmatrix}$$

(4) **(2 points)** Let

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}.$$

(a) Find the eigenvalues of A.

Sol:

$$\begin{vmatrix} 1-\lambda & 2 \\ 3 & 2-\lambda \end{vmatrix} = (\lambda - 1)(\lambda - 2) - 6 = \lambda^2 - 3\lambda - 4 = (\lambda - 4)(\lambda + 1).$$

So, the eigenvalues are $\lambda = 4$ and $\lambda = -1$.

(b) Find the eigenspace of A.

Sol:

Let $\lambda = 4$. Then

$$\begin{bmatrix} -3 & 2 \\ 3 & -2 \end{bmatrix} \implies \text{eigenvector is } \begin{bmatrix} 2 \\ 3 \end{bmatrix}.$$

Let $\lambda = -1$. Then

$$\begin{bmatrix} 2 & 2 \\ 3 & 3 \end{bmatrix} \implies \text{eigenvector is } \begin{bmatrix} -1 \\ 1 \end{bmatrix}.$$